

# The 3-D Inverted Pendulum with Oscillating Pivot

---

*By: Mr. A. Ohev-Zion, Emeritus. B.Z. Sandler and Dr. A. Shapiro.*

When we consider the ways we can balance an inverted pendulum at its upper unstable equilibrium point, we intuitively consider the ways we can move its base along the horizontal axis. The traditional known (and more associative) ways are: force a torque on the pendulum around its pivot, place the pivot upon a moving cart or, combine the first 2 by placing the pivot at the center of a wheel and force a torque between the pendulum and the wheel.

The method we discuss here is less intuitive and therefore more interesting, and it's a known method since the last century. An inverted pendulum can be dynamically balanced at its unstable upper equilibrium point by rapidly oscillating its pivot along the vertical axis, in small amplitude.

In this article we discuss the use of the last method to dynamically stabilize a 3-Dimensional inverted pendulum. The inverted pendulum has 2 perpendicular pivots, allowing it to tilt out of the vertical plane into a 3-D angle.

To achieve this, we bring the equations describe the pendulum's dynamics to a non-dimensional form. Then we calculate analytically and numerically the influence of the pendulum's parameters on its dynamic stability.

We find the close relation between the pendulum height, to the oscillations amplitude and frequency. We also perform several experiments and compare the results to the theory.